Mathematical Modeling and Analysis

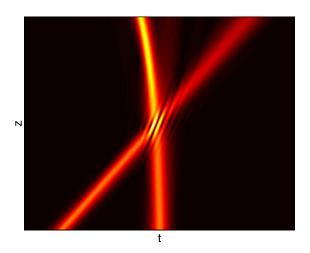


Effects of Stimulated Raman Scattering on Multichannel Soliton Transmission

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The intrinsic robustness of optical solitons makes them ideal candidates as bit carriers in high speed communication systems. However, the need for increased transmission capacity pushes development in two directions: the expansion to multichannel transmission and the use of ultrashort pulses as bit carriers. Because of dispersion in optical fibers, bitstreams in different frequency channels propagate at different speeds. Thus a bitstream in one channel will catch up with one in another channel and a collision ensues. When solitons are used as bit carriers and high order effects are absent, this poses no threat to information transmission since solitons pass through one another without changing. But as the bit rate in each channel increases, the pulse duration must decrease and high order effects begin to influence pulse propagation and collisions. For example, solitons no longer pass through one another transparently. Rather, they undergo a change in their characteristic parameters, which may include amplitude, frequency, phase, and/or position. Furthermore, they lose energy through the emission of radiation when they collide with one another. After many collisions (which is a routine occurrence in a real world system) these effects could decrease system performance by causing, for example, the amplitude of a pulse to drop below the threshold for detecting a logical one, resulting in the detection of an erroneous logical zero. The research performed during the summer of 2004 focuses on the effects of stimulated Raman scattering, which is a leading order effect in ultrashort pulse propagation [1], on soliton collisions.

Stimulated Raman scattering occurs when pho-



Numerical simulation of a two soliton collision in the presence of stimulated Raman scattering. Note the self frequency shift causes both pulses to curve to the left, and that after the collision, the zero-channel pulse gains energy while the β -channel pulse looses energy.

tons in a pulse excite the medium through which they are propagating. The molecules making up the medium remain in this excited state for some time before relaxing to a lower energy level, thereby coherently emitting a photon back into the pulse. Due to the different orientations of and stresses on these molecules, the photons which are emitted back into the pulse do not have the same frequency as those that initially excited the molecules. The net result of this process is a continuous downshift in soliton frequency. Pulse dynamics in the presence of stimulated Raman scattering are governed by the following modified nonlinear Schrödinger equation:

$$iu_z + u_{tt} + 2|u|^2 u = -\varepsilon(|u|^2)_t u,$$
 (1)

where u is the complex electric field envelope, z is propagation distance, and t is time measured in the reference frame which is traveling at the group velocity of the pulse. In this analysis, the Raman coefficient ε is assumed small, which is consistent with applications in telecommunication systems. This assumption also permits a perturbative analysis of the problem.

To study the Raman induced effects on soli-

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ton collisions, we first study a collsion between two solitons in different frequency channels. The ansatz

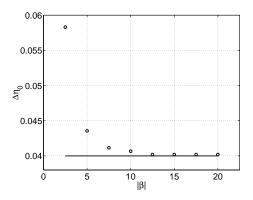
$$u = u_0 + u_\beta + v_0 + v_\beta,$$

consisting of solitons u_0 and u_β and collision induced effects v_0 and v_β in the zero and β frequency channels, is plugged into Equation (1) and the resonant approximation is made, resulting in equations for v_0 and v_β with two small parameters ε and $|\beta|^{-1}$, the reciprocal of the interchannel frequency difference. These are then projected onto the eigenmodes of the linear operator describing the dynamics about the soliton solution [2] to determine which soliton parameters are effected by the collision. This double perturbation theory obtains the following expressions for collision induced amplitude shift and cross-frequency shift of the zero- and β -channel solitons:

$$\begin{split} \Delta\eta_0 &= 2\epsilon sgn(\beta)\eta_0\eta_\beta & \Delta\eta_\beta &= -2\epsilon sgn(\beta)\eta_0\eta_\beta \\ \Delta\beta_0 &= -\frac{8\epsilon\eta_0^2\eta_\beta}{3|\beta|} & \Delta\beta_\beta &= -\frac{8\epsilon\eta_0\eta_\beta^2}{3|\beta|} \end{split}$$

The amplitude shift on the zero-channel soliton is different only by a sign from that of the β -channel soliton. This is consistent with the fact that Equation (1) conserves energy. However, the governing equation does not conserve momentum, which is reflected in the fact that the cross frequency shift of both the zero- and β -channel solitons have the same sign.

These results are verified to excellent agreement using numerical simulations. The numerical method uses a fourth-order split-step technique [3] in which the linear term in Equation (1) is integrated exactly in the frequency domain (obtained via the fast Fourier transform) and the nonlinear terms are integrated using a fourth-order Runge-Kutta algorithm. The periodic boundary conditions of the fast Fourier transform allow radiation emitted during collisions to re-enter the collision region of the computational domain, resulting in spurious, unphysical results. To suppress this effect, absorbing boundary conditions are used. This method is shown to be $O(\Delta z^4, e^{-1/\Delta t})$ accurate, to conserve energy, and



Comparison of theory (line) and numerics (dots) for the amplitude shift as a function of interchannel frequency difference $\Delta\eta_0(|\beta|)$ for the case when $\eta_0=1=\eta_\beta$ and $\epsilon=0.02$. The theory improves as $|\beta|$ increases.

to reproduce analytical results on self frequency shift [4].

The main goal of this project is to determine the effects of stimulated Raman scattering on soliton collisions while taking the stochastic nature of bitstreams into account. Recent research [5] reveals that both the pulse amplitude and frequency are lognormally distributed when the pulse collides with a bitstream in a different frequency channel. Usually such parameters are assumed to be Gaussian distributed, but this is a very poor approximation to the lognormal distribution. In fact, this analysis shows that the collision induced effects of stimulated Raman scattering are much stronger and have a more malign effect on system performance than previously thought. This project will continue by using the numerical method described above to verify the analytical results obtained in [5].

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